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ABSTRACT

Charged-particle energy loss or “stopping power” in plasmas has been studied theoretically and experimentally, with important applications in modeling fusion experiments. Dense plasmas relevant to inertial fusion are theoretically challenging, but several models have been developed. Here, we report several physically motivated modifications to the parameterization of the Li-Petrasso stopping-power model. The new parameterization described in this work leads to larger discrepancies between the Li-Petrasso model and both other theories and experimental data near the Bragg peak for plasma stopping, corroborating recent conclusions that the Li-Petrasso model is not accurate in this regime [Frenje *et al.*, Phys. Rev. Lett. **122**, 015002 (2019)]. Conversely, our modified parameterization agrees better with other theories in the high-velocity limit.

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I. INTRODUCTION

The basic principle of laboratory fusion is to use the energy produced from nuclear reactions to self-heat the plasma, counteracting loss mechanisms and resulting in a sustained controlled burn or a rapidly increasing temperature and energy production. Magnetic fusion schemes operate with low density plasmas that have a large Coulomb logarithm and can be treated like classical plasmas. Inertial confinement fusion (ICF) systems, in contrast, operate with very high-density plasmas that have small Coulomb logarithms and that are potentially cool enough to be strongly coupled and electron degenerate, such as in the dense fuel of a typical hot-spot design.¹

Modeling fusion self-heating in these systems requires a theory of the rate at which an energetic charged particle loses energy to the plasma, as the dominant fusion self-heating mechanism is by the DT fusion’s α product, initially at 3.5 MeV. In this regime, large-angle Coulomb scattering can play a critical role, especially toward the end of the α range, resulting in a difference between the linear and path length energy transfer.² α transport in an ICF hot spot is important for calculating the hot-spot energy balance and dynamical evolution;³ the thresholds for burning plasma and ignition regimes are sensitive to the choices of theoretical models used.^{4,5} Stopping powers are also relevant to ICF diagnostics that employ charged particles including radiography^{6,7} and spectroscopy.^{8–13} In these applications, the linear and path length energy transfer are similar since scattering effects are

insignificant. As an example, in Ref. 12, uncertainty in the stopping-power model is a significant source of systematic uncertainty in the inferred areal density (ρR). Experiments studying fusion-product spectra from nuclear reactions occurring in implosions,¹⁴ with applications to basic nuclear physics and nuclear astrophysics, may need to correct for energy loss.^{15,16}

Many theories of charged-particle energy loss applicable to ICF modeling have been developed in the last three decades, notably the Maynard-Deutsch (MD),^{17,18} Li-Petrasso (LP),^{19,20} and Brown-Preston-Singleton (BPS)²¹ theories, which are widely used. Theories based on the Lenard-Balescu kinetic equation include weak collisions and plasmon excitations, an example being MD. Theories based on convergent kinetic theory, such as BPS, combine both Lenard-Balescu and Boltzmann physics, including both collective excitations and strong collisions, but we note that the BPS theory does not include a treatment of quantum degeneracy effects. In general, stopping approaches derived from the Boltzmann equation are expected to be accurate in the low-velocity (strong scattering) regime, while models based on the Lenard-Balescu equation are expected to be accurate in the high-velocity (weak scattering) regime.

LP was the first theory to cover both ranges of applicability by including an approximate application of both approaches. The LP model was the first to treat a generalized Fokker-Planck equation for moderately coupled plasmas,²² which results in a stopping-power

evaluation including terms proportional to $1/\ln \Lambda$ (where $\ln \Lambda$ is the Coulomb logarithm), an important development since many ICF-relevant plasmas have $\ln \Lambda < 10$. Contributions from large-angle Coulomb scattering are thus treated, which enhances the linear stopping power since the effective path length per linear distance increases, an effect which is also related to energy straggling and blooming. We note that the Fokker-Planck equation includes a number of approximations and is not as exact as other approaches to the kinetic theory including Boltzmann and Lenard-Balescu approaches.²³ The lack of collective excitations in the Fokker-Planck equation leads to Li and Petrasso introducing a second term to account for plasmons. The resulting expression is also modified to account for the quantum effects on scattering and the effective temperature through a degeneracy correction. The LP theory is broadly used in ICF research due to its conceptual and computational simplicity. BPS is another combined theory, based on convergent kinetic theory.

In addition to theory, new experimental measurements are being reported, including in warm-dense matter and ICF-relevant plasmas for the first time. Recent reports have been published of fast particles stopping in warm-dense matter plasmas (moderately coupled and degenerate),^{24,25} in ICF hot-spot relevant plasmas generated by implosions,^{26–28} and in novel laser-generated plasmas probed by accelerator beams.^{29,30} These experiments span a wide range of plasma conditions, but the probing particles lose a relatively small fraction of their initial energy. A proper and well-documented treatment of the theory is necessary for comparison to the data and other theories.

In this work, we discuss several modifications and generalizations to the original LP theory, specifically the parameterizations of physical quantities, and compare to the MD and BPS theories. Some of these modifications are already being used by the community and this paper serves, in part, to document the various choices that we recommend using in evaluating the LP model. The theoretical discussion of LP is given in Sec. II. Our modified LP results are compared to the originally published LP theory and MD plus BPS in Sec. III, and this paper is concluded in Sec. IV.

II. MODIFIED LI-PETRASSO FORMALISM

The published LP stopping power is expressed as

$$\frac{dE}{dx} = - \left(\frac{Z_t e}{v_t} \right)^2 \omega_{pf}^2 \left\{ G(x^{t/f}) \ln \Lambda_b + \Theta(x^{t/f}) \ln \left(1.123 \sqrt{x^{t/f}} \right) \right\}, \quad (1)$$

see Eq. (3) in Ref. 19. Here, Z_t is the test particle charge in atomic units, e is the fundamental charge, v_t is the velocity, and ω_{pf} is the plasma frequency for field particle species f (defined $\omega_{pf} = \sqrt{4\pi n_f e_f^2 / m_f}$, where e_f is the field particle charge, m_f is the mass, and n_f is the particle number density). The parameter $x^{t/f}$ is a dimensionless ratio of the test-to-field particle velocity, which will be discussed later. In plasmas with multiple species (e.g., electrons and ions), the total stopping power is

$$\frac{dE}{dx} = \sum_f \frac{dE}{dx_f}. \quad (2)$$

The prefactor in Eq. (1) gives the overall normalization. Inside the brackets, $G(x^{t/f}) \ln \Lambda_b$ represents the stopping power from binary Coulomb collisions, and $\Theta(x^{t/f}) \ln \left(1.123 \sqrt{x^{t/f}} \right)$ represents slowing due to collective plasma effects.

A. Velocity ratio $x^{t/f}$

As clarified in the recent Erratum (Ref. 19), the original LP paper gave only one expression for the parameter $x^{t/f}$, inadvertently implying that it was equivalent in the binary and collective terms. In the collective term, the parameter $\sqrt{x^{t/f}}$ is in fact $v_t / \omega_{pf} \lambda_{Df}$ (see Ref. 31), where λ_{Df} is the Debye length ($= \sqrt{k_B T_f / 4\pi n_f e_f^2}$). Thus, in the collective effects term, $x^{t/f} = v_t^2 m_f / k_B T_f$, while in the binary collision term, $x^{t/f} \equiv v_t^2 / v_f^2$ with $v_f^2 = 2k_B T_f / m_f$, so $x^{t/f} = v_t^2 m_f / 2k_B T_f$. Relative to the original publication, the LP erratum change the collective effects term by $\sqrt{2}$ inside the logarithm, which can be significant when the collective stopping is an appreciable fraction of the total. From here on, we use $x^{t/f}$ only in the binary collision term, i.e., with $x^{t/f} \equiv v_t^2 / v_f^2$ with $v_f^2 = 2k_B T_f / m_f$, and write the collective effects stopping in terms of $v_t / \omega_{pf} \lambda_{Df}$ for clarity.

Further, the form of the collective effects term published in Ref. 19 was derived under the assumption that $x^{t/f} \gg 1$ (see Ref. 31), but the regime $x^{t/f} \sim 1$ is important for hot-spot self-heating in ICF, where the α stopping often occurs near the “Bragg peak.” A more general form of Eq. (1) can be written as

$$\begin{aligned} \frac{dE}{dx} = & - \left(\frac{Z_t e}{v_t} \right)^2 \omega_{pf}^2 \left\{ G(x^{t/f}) \ln \Lambda_b + \left(\frac{\omega_{pf} \lambda_{Df}}{v_t} \right) \right. \\ & \left. \times K_0 \left(\frac{\omega_{pf} \lambda_{Df}}{v_t} \right) K_1 \left(\frac{\omega_{pf} \lambda_{Df}}{v_t} \right) \right\}, \quad (3) \end{aligned}$$

where K_0 and K_1 are the irregular modified cylindrical Bessel function of the zeroth and first order, respectively. This expression places no limits on the values of $x^{t/f}$.

The factor G in the binary collision term is sometimes referred to as the Chandrasekhar function, which as modified in Ref. 19 to include $1/\ln \Lambda$ terms is given by

$$G(x^{t/f}) = \mu(x^{t/f}) - \frac{m_f}{m_t} \left\{ \frac{d\mu(x^{t/f})}{dx^{t/f}} - \frac{1}{\ln \Lambda_b} \left[\mu(x^{t/f}) + \frac{d\mu(x^{t/f})}{dx^{t/f}} \right] \right\}, \quad (4)$$

where

$$\mu(x^{t/f}) = \frac{2}{\sqrt{\pi}} \int_0^{x^{t/f}} e^{-\xi} \sqrt{\xi} d\xi, \quad (5)$$

$$= \frac{\sqrt{\pi}}{2} \operatorname{erf} \left(\sqrt{x^{t/f}} \right) - \sqrt{x^{t/f}} e^{-x^{t/f}}, \quad (6)$$

$$\frac{d\mu(x^{t/f})}{dx^{t/f}} = \sqrt{x^{t/f}} e^{-x^{t/f}}. \quad (7)$$

As noted previously, G is evaluated where $x^{t/f} = v_t^2 / v_f^2$ and $v_f^2 = 2k_B T_f / m_f$. Here, erf is the error function.

B. Coulomb logarithm

The Coulomb logarithm, $\ln \Lambda_b$, in the Li-Petrasso paper is given by

$$\ln \Lambda_b = \ln \left(\frac{\lambda_D}{p_{min}} \right), \quad (8)$$

which is an approximation by Trubnikov³² in the limit $\lambda_D \gg p_{min}$. A more general form is given by

$$\ln \Lambda_b = \frac{1}{2} \ln \left[1 + \left(\frac{\lambda_D}{p_{min}} \right)^2 \right]. \quad (9)$$

While Eq. (9) will reduce to Eq. (8) in the appropriate limit, the approximation by Trubnikov is not valid for all ICF-relevant scenarios. For example, if $\lambda_D \sim 3p_{min}$, then Eqs. (9) and (8) differ by about 5%.

For the Coulomb logarithm, Li and Petrasso use the electron Debye length (λ_{De}). For stopping on electrons, we also use $\lambda_D = \lambda_{De}$. However, for ion stopping, the Debye length is taken as the total Debye length for all species in the plasma, as it represents the absolute screening length,

$$\lambda_D = \frac{1}{\sqrt{\sum_f 4\pi n_f e_f^2 / k_B T_f}}. \quad (10)$$

This is particularly motivated by plasmas with a minority ion species. In that scenario, if the Debye length were evaluated for only the minority, it would be significantly longer than physical. Potential applications include fusion product transport in mixed plasmas.³³ We note that plasma ion stopping, and electronic stopping near the Bragg peak, are typically dominated by strong scattering and care must be taken applying theories in this regime.²¹

C. Relative velocity u

The term p_{min} in Eqs. (8) and (9) corresponds to a minimum impact parameter (maximum momentum transfer) and is given by Li-Petrasso as

$$p_{min} = \sqrt{p_{\perp}^2 + \left(\frac{\hbar}{2m_r u} \right)^2}, \quad (11)$$

where $p_{\perp} = e_f e_f / m_r u^2$ is the classical impact parameter, and the second term represents maximum momentum transfer in the regime where the quantum de Broglie wavelength may be greater than the classical minimum impact parameter. In this equation, m_r is the reduced mass of test and field particles and u is a relative velocity between test and field particles defined as $\vec{u} = |\vec{v}_t - \vec{v}_f|$. We suspect that the approximation $u = \sqrt{v_t^2 + v_f^2}$ is used in many evaluations of the LP dE/dx in the literature, which would be accurate only when $v_t \gg v_f$ or $v_f \gg v_t$, i.e., not when $v_t \sim v_f$, near the Bragg peak. u can be rigorously derived by integrating the complete Maxwellian field particle distribution in three dimensions,

$$u \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sqrt{(v_t - v_x)^2 + v_y^2 + v_z^2} A e^{-m_f v^2 / 2k_B T_f} dv_x dv_y dv_z, \quad (12)$$

where A is the normalization of the distribution function and v_t is the test particle velocity, which is taken as $\vec{v}_t \parallel \hat{x}$ without loss of generality. In the exponent, $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$, it is relatively straightforward to directly evaluate the integral of the distribution function, which results in the expression

$$u = \sqrt{\frac{2k_B T_f}{\pi m_f}} \exp \left[-\frac{m_f v_t^2}{2k_B T_f} \right] + v_t \left(1 + \frac{k_B T_f}{m_f v_t^2} \right) \operatorname{erf} \left[\sqrt{\frac{m_f v_t^2}{2k_B T_f}} \right]. \quad (13)$$

Since Eq. (13) is simple to evaluate numerically, we recommend its use in all but the most demanding applications.

When the test particle is slow ($v_t \ll v_{th}$),

$$\sqrt{(v_t - v_x)^2 + v_y^2 + v_z^2} \rightarrow \sqrt{v_x^2 + v_y^2 + v_z^2} = v, \quad (14)$$

which means that the relative velocity reduces to

$$u = \int_0^{\infty} v A e^{-mv^2/2k_B T_f} dv = \sqrt{\frac{8k_B T_f}{\pi m_f}}. \quad (15)$$

In the high-velocity limit ($v_t \gg v_{th}$),

$$\sqrt{(v_t - v_x)^2 + v_y^2 + v_z^2} \rightarrow v_t, \quad (16)$$

and the relative velocity reduces to

$$u = v_t. \quad (17)$$

For intermediate values, one can construct a “simple relative velocity” from these two results, rather than evaluating the integral over the distribution function. This velocity is given by

$$u_s = \sqrt{v_t^2 + \left(\frac{8k_B T_f}{\pi m_f} \right)^2}. \quad (18)$$

When $v_t \sim v_{th}$, this expression has an error of about 2.5% compared to the exact expression [Eq. (13)]. Generally, this is acceptable since u only appears in the Coulomb logarithm.

D. Electron degeneracy

In degenerate regimes where the Fermi energy can be comparable to the thermal temperature, it is common to modify the plasma temperature to account for quantum degeneracy, which was noted in Ref. 19 but without an explicit prescription for an effective temperature. Reference 2 gives an expression for an “effective temperature” (T_{eff}) in terms of the Fermi temperature (T_F),

$$T_{eff} = \frac{3}{5} T_F \left[1 + \frac{5\pi^2}{12} \left(\frac{T_e}{T_F} \right)^2 - \frac{\pi^4}{16} \left(\frac{T_e}{T_F} \right)^4 + \dots \right], \quad (19)$$

which is only valid in the strong-degeneracy regime $T_e \ll T_F$. This problem is also discussed in Ref. 34. Separately, given in Ref. 31, in the weak degeneracy regime, the effective energy per electron is given as

$$E_{eff} = \frac{3}{2} k_B T_e \left[1 + \frac{1}{2^{5/2}} \frac{4}{3\sqrt{\pi}} \left(\frac{E_F}{k_B T_e} \right)^{3/2} - \frac{1}{3^{5/2}} \left(\frac{4}{3\sqrt{\pi}} \right)^2 \left(\frac{E_F}{k_B T_e} \right)^3 \dots \right], \quad (20)$$

with $T_{eff} = 2E_{eff}/3k_B$. To combine the two regimes, strong and weak degeneracy, Ref. 31 uses

$$\frac{1}{E_{eff}} = \sqrt{\frac{1}{E_{eff,weak}^2} + \frac{1}{E_{eff,strong}^2}}. \quad (21)$$

Here, we use an effective field particle temperature correction following Drake.³⁵ The total pressure (p_e) of quasidegenerate electron fluid may be written as [Eq. (3.22) in Ref. 35]

$$p_e = \frac{2}{3} n_e k_B T_e \frac{F_{3/2}\left(\frac{\mu_e}{k_B T_e}\right)}{F_{1/2}\left(\frac{\mu_e}{k_B T_e}\right)}, \quad (22)$$

which leads to an expression for T_{eff}

$$T_{eff} = T_e \frac{F_{3/2}\left(\frac{\mu_e}{k_B T_e}\right)}{F_{1/2}\left(\frac{\mu_e}{k_B T_e}\right)}. \quad (23)$$

In this expression, $F_{1/2}$ and $F_{3/2}$ are the Fermi integrals,

$$F_j(x) = \frac{1}{\Gamma(j+1)} \int_0^\infty dt \frac{t^j}{\exp(t-x) + 1}, \quad (24)$$

and μ_e is the chemical potential, which can be derived from the fit by Drake [Eq. (3.20) in Ref. 35]

$$\frac{\mu_e}{k_B T_e} = -\frac{3}{2} \ln \Theta + \ln \left(\frac{4}{3\sqrt{\pi}} \right) + \frac{0.25054\Theta^{-1.858} + 0.072\Theta^{-1.858/2}}{1 + 0.25054\Theta^{-0.868}}, \quad (25)$$

where Θ is the ratio of temperature to Fermi temperature, given by

$$\Theta = \frac{T_e}{T_F} = T_e \left[\left(\frac{8\pi}{3n_e} \right)^{2/3} \frac{2m_e k_B}{h^2} \right]. \quad (26)$$

In the nondegenerate limit of large Θ , Eq. (23) gives $T_{eff} \rightarrow T_e$ and smoothly transitions to the degenerate regime. We note that this simple approximation is likely accurate only in the weakly degenerate regime and care must be taken in the strongly degenerate regime, as T_{eff} becomes less representative and other parts of the problem, such as the relative velocity u are affected.

E. Summary

To express the modified Li-Petrasso formalism, we use Eq. (3) with the modified Chandrasekhar function defined by Eq. (4), and the Coulomb logarithm calculated from Eqs. (9), (10), (11), and (13) with a quantum correction given by Eqs. (22)–(26).

III. COMPARISON

A comparison to the published version of the theory is shown in Fig. 1. In most regimes, the significant changes are to the collective effects stopping term, specifically the factor of 2 error as noted in Ref. 20 and Sec. II A. We thus evaluate the LP stopping model in three ways:

- LP # 1: LP as published in 1993, i.e., Eq. (1) with $x^{t/f} = v_i^2 m_f / 2k_B T_f$ in all terms.

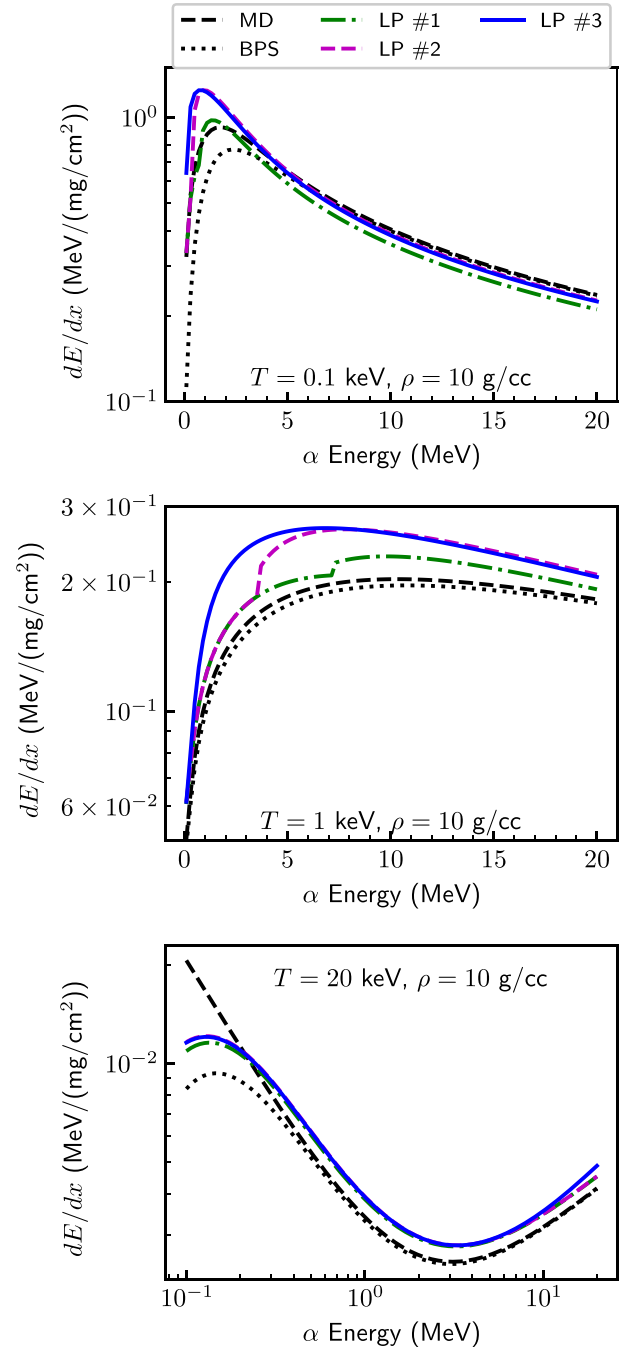


FIG. 1. Three evaluations of the LP stopping power (see the text) compared to MD and BPS for α s stopping in a DT plasma at 10 g/cc and 0.1 (top), 1 (center), and 20 keV (bottom).

- LP # 2: LP Erratum (Ref. 20), i.e., Eq. (1) but with $x^{t/f}$ properly treated in the binary and collective terms, i.e., with the latter replaced by $v_i/\omega_{pf} \lambda_D$.
- LP # 3: this model, i.e., Eq. (3).

While the evaluation # 1 is superseded by Ref. 20, we include it to demonstrate the magnitude of the correction. LP # 1 and # 2 use the collective effects term with a step function [i.e., Eq. (1)]. All evaluations use the modified Coulomb logarithm [Eq. (9)] and Debye length as prescribed in Sec. II B, plus the relative velocity and effective temperature from Secs. II C and II D, respectively. In Fig. 1, these models are evaluated for α particles stopping in a DT plasma at temperatures of 0.1, 1, and 20 keV and a density of 10 g/cc. These comparisons focus on electronic stopping due to its relevance to recent experiment results, but in the evaluation in 20 keV, the ionic stopping dominates at low energies. The major effect is that the changes to the collective stopping treatment increase dE/dx , especially noticeable near the Bragg peak and in the high-velocity limit. In contrast, the low-velocity stopping region, including ionic stopping (see the bottom panel of Fig. 1), is insensitive to the differences between these parameterizations, largely because the collective effects are negligible. The LP model as published in 1993, including the incorrect treatment of $x^{1/f}$, surprisingly, is the closest to the MD and BPS models around the Bragg peak, while the modified LP stopping power is closer to MD and BPS in the high-velocity limit. The fact that LP evaluated with the incorrect treatment of $x^{1/f}$ is closer to MD and BPS near the Bragg peak may result from the simple dielectric collective effects term used in LP overcontributing to the total.

Figure 2 compares the modified LP stopping in this paper to a number of models over a range of relevant parameters for proton stopping. In addition to BPS and MD, we include Grabowski’s classical molecular dynamics results,³⁶ and cold-matter tabular stopping power

from the SRIM code³⁷ in the first (and lowest temperature) case. These models are evaluated for protons in the following:

- (a) Solid-density warm-dense-matter beryllium (relevant to Ref. 24)
- (b) A D^3He plasma at the modest density and temperature roughly relevant to Ref. 26
- (c) In a CH plasma at a high density and low temperature, relevant to stopping in imploding shells (Refs. 12 and 13)
- (d) In an ignition-relevant hot spot

In the lower-density examples (a) and (b), the modified LP model agrees well with MD and BPS in the high-velocity limit while predicting a higher stopping power around the Bragg peak. Significant differences are observed between the models at a higher density (c) and (d).

We directly compare theories over a wide range of parameters in Fig. 3. The plot shows the ratio of LP to MD theory (top row) and LP to BPS (bottom row) for D^3He protons (left) and DT α s (right) over several orders of magnitude in density and temperature. The LP and MD theories disagree around the Bragg peak, with the Li-Petrasso theory giving a higher stopping power by around 20%–30%. This causes the vertical bands at 20–30 keV for D^3He protons, and around 1–2 keV for the DT- α s. Similarly, we observe a discrepancy near the Bragg peak between LP and BPS. At a high density and a modest or low temperature (upper left of the plots), all of the three theories disagree substantially, by factors of $2\times$ or more. We note this is a challenging regime with strong coupling and degeneracy effects, and the BPS theory, in particular, does not include an effective temperature correction for electron-degenerate plasmas or strong-coupling effects.

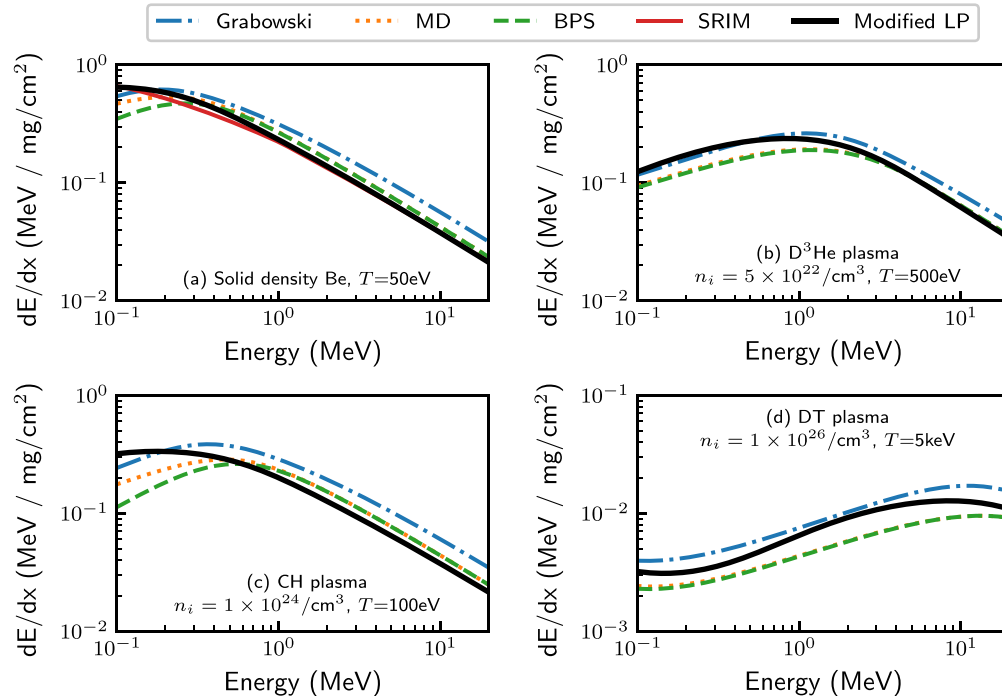


FIG. 2. Comparison of several stopping theories for protons slowing in various ICF-relevant plasmas. (a) Fully ionized solid-density Be at 50 eV [Warm Dense Matter (WDM) regime], (b) in a D^3He plasma at 500 eV and $5 \times 10^{22}/cm^3$, (c) in a CH plasma at $10^{24}/cm^3$ and 100 eV, relevant to dense shells, and (d) in a high-density and hot DT plasma at 5 keV and $10^{26}/cm^3$, relevant to ignition targets.

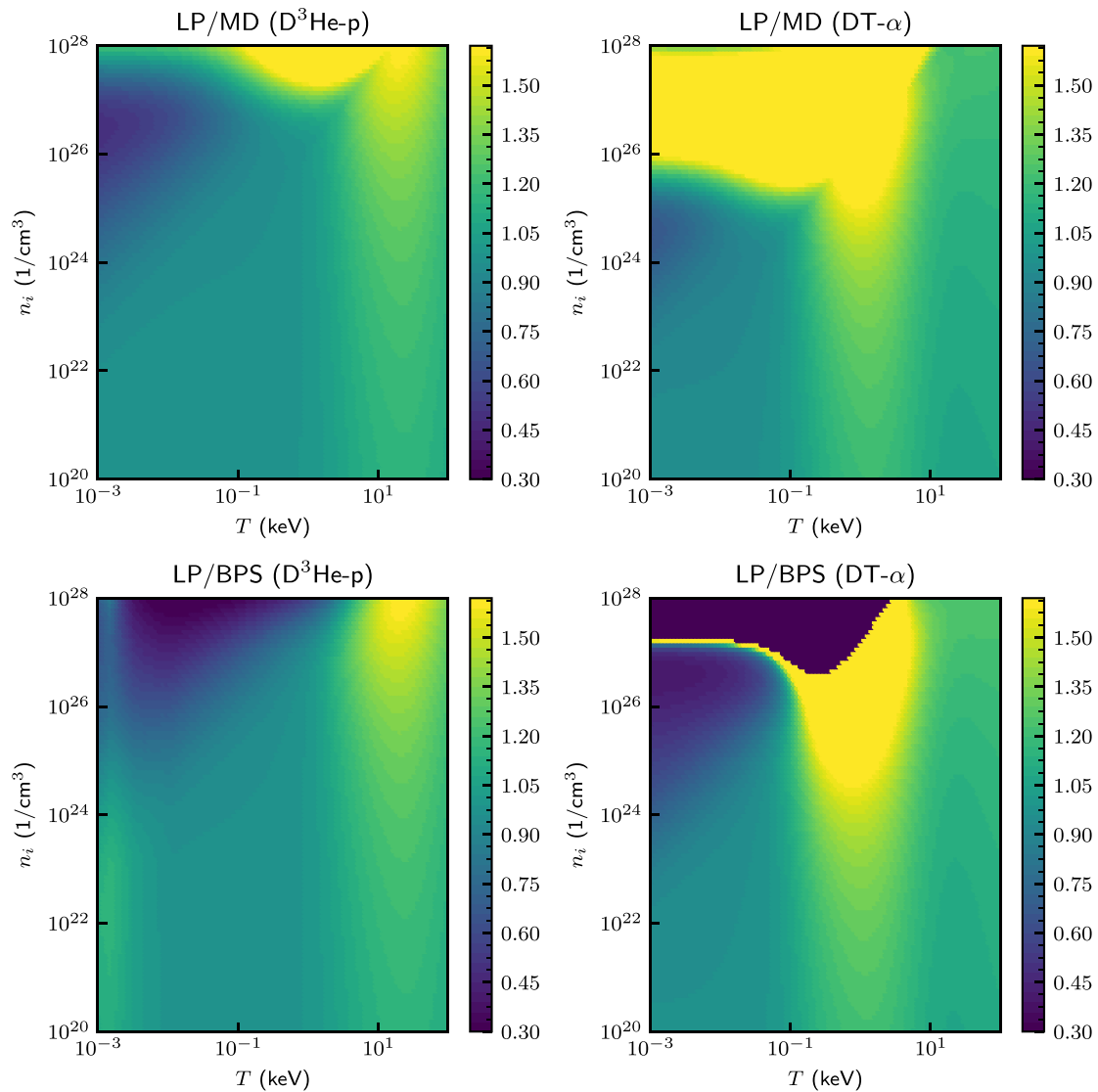


FIG. 3. Ratio of stopping powers for $D^3\text{He-p}$ (14.7 MeV, left) and $\text{DT-}\alpha$ (3.5 MeV, right) slowing in a DT plasma at various densities and temperatures ($T = T_e = T_i$). Top row: ratio of LP and MD theories and bottom row: ratio of LP and BPS.

Across these comparisons, we only consider the instantaneous stopping power, and a more detailed investigation of the impact of scattering effects is not considered here.

IV. CONCLUSION

In this paper, we discuss several modifications to the parameterization of the LP stopping-power theory that are either physically motivated or used to extend the range of applicability. Some of these modifications are already in use in evaluations or computational implementations but have not been described previously in the literature. They include a modified treatment of the collective-effects stopping power (Sec. II A), a more general form of the Coulomb logarithm (Sec. II B), a physical treatment of the relative velocity u (Sec. II C),

and a generally applicable effective temperature for electron degenerate plasmas (Sec. II D). This modified LP theory can be used in modeling, diagnosing, and understanding ICF and high-energy-density plasmas. The modified theory exhibits a higher stopping power near the Bragg peak than the originally published LP theory and other theories such as MD and BPS, typically by 20%–30%, while the high-velocity limit is in better agreement with MD and BPS than the original LP. This is attributed primarily to the modified collective effects treatment.

Based on this work, we conclude that the higher stopping power calculated near the Bragg peak in the modified LP theory is physically motivated based on the derivation and inputs to the model, and therefore, measurements near the Bragg peak should be able to distinguish

between the LP theory and other models such as MD and BPS. These results therefore corroborate the interpretation of recently published experimental results, especially those of Cayzac *et al.*,³⁰ Frenje *et al.*,²⁷ and Sayre *et al.*,²⁸ which found that the LP model overestimates dE/dx near the Bragg peak by tens of percent when the particles lose a small fraction of their initial energy, and we expect other theories, notably MD and BPS, to be more accurate in this regime. In other regimes, especially the high-velocity limit for low or modest density plasmas, the modified LP theory agrees well with MD and BPS. The effect on the total range of charged particles, which is particularly important for fusion self-heating, has an additional complication from scattering effects, which is worthy of additional study both theoretically and experimentally. The theoretical study of charged-particle stopping power is also an active area of study, and we encourage work including *ab initio* simulations using techniques like time-dependent density-functional theory to estimate the accuracy and range of validity for simple stopping-power theory.

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REFERENCES

- J. Nuckolls, L. Wood, A. Thiessen, and G. Zimmerman, *Nature* **239**, 139–142 (1972).
- C. Li and R. D. Petrasso, *Phys. Plasmas* **2**, 2460–2464 (1995).
- O. A. Hurricane, P. T. Springer, P. K. Patel, D. A. Callahan, K. Baker, D. T. Casey, L. Divol, T. Döppner, D. E. Hinkel, M. Hohenberger, L. F. Berzak Hopkins, C. Jarrott, A. Kritcher, S. Le Pape, S. Maclaren, L. Masse, A. Pak, J. Ralph, C. Thomas, P. Volegov, and A. Zylstra, *Phys. Plasmas* **26**, 052704 (2019).
- M. Temporal, B. Canaud, W. Cayzac, R. Ramis, and R. L. Singleton, *Eur. Phys. J. D* **71**, 132 (2017).
- A. B. Zylstra and O. A. Hurricane, *Phys. Plasmas* **26**, 062701 (2019).
- C. K. Li, F. H. Séguin, J. A. Frenje, J. R. Rygg, R. D. Petrasso, R. P. J. Town, P. A. Amendt, S. P. Hatchett, O. L. Landen, A. J. Mackinnon, P. K. Patel, V. A. Smalyuk, J. P. Knauer, T. C. Sangster, and C. Stoeckl, *Rev. Sci. Instrum.* **77**, 10E725 (2006).
- A. B. Zylstra, C. K. Li, H. G. Rinderknecht, F. H. Séguin, R. D. Petrasso, C. Stoeckl, D. D. Meyerhofer, P. Nilson, T. C. Sangster, S. Le Pape, A. Mackinnon, and P. Patel, *Rev. Sci. Instrum.* **83**, 013511 (2012).
- F. H. Séguin, J. A. Frenje, C. K. Li, D. G. Hicks, S. Kurebayashi, J. R. Rygg, B.-E. Schwartz, R. D. Petrasso, S. Roberts, J. M. Soares, D. D. Meyerhofer, T. C. Sangster, J. P. Knauer, C. Sorce, V. Y. Glebov, C. Stoeckl, T. W. Phillips, R. J. Leeper, K. Fletcher, and S. Padalino, *Rev. Sci. Instrum.* **74**, 975–995 (2003).
- F. H. Séguin, N. Sinenian, M. Rosenberg, A. Zylstra, M. J.-E. Manuel, H. Sio, C. Waugh, H. G. Rinderknecht, M. G. Johnson, J. Frenje, C. K. Li, R. Petrasso, T. C. Sangster, and S. Roberts, *Rev. Sci. Instrum.* **83**, 10D908 (2012).
- A. B. Zylstra, J. A. Frenje, F. H. Séguin, M. J. Rosenberg, H. G. Rinderknecht, M. G. Johnson, D. T. Casey, N. Sinenian, M. J.-E. Manuel, C. J. Waugh, H. W. Sio, C. K. Li, R. D. Petrasso, S. Friedrich, K. Knittel, R. Bionta, M. McKernan, D. Callahan, G. W. Collins, E. Dewald, T. Döppner, M. J. Edwards, S. Glenzer, D. G. Hicks, O. L. Landen, R. London, A. Mackinnon, N. Meezan, R. R. Prasad, J. Ralph, M. Richardson, J. R. Rygg, S. Sepke, S. Weber, R. Zacharias, E. Moses, J. Kilkenny, A. Nikroo, T. C. Sangster, V. Glebov, C. Stoeckl, R. Olson, R. J. Leeper, J. Kline, G. Kyrala, and D. Wilson, *Rev. Sci. Instrum.* **83**, 10D901 (2012).
- M. J. Rosenberg, F. H. Séguin, C. J. Waugh, H. G. Rinderknecht, D. Orozco, J. A. Frenje, M. G. Johnson, H. Sio, A. B. Zylstra, N. Sinenian, C. K. Li, R. D. Petrasso, V. Y. Glebov, C. Stoeckl, M. Hohenberger, T. C. Sangster, S. LePape, A. J. Mackinnon, R. M. Bionta, O. L. Landen, R. A. Zacharias, Y. Kim, H. W. Herrmann, and J. D. Kilkenny, *Rev. Sci. Instrum.* **85**, 043302 (2014).
- A. B. Zylstra, J. A. Frenje, F. H. Séguin, D. G. Hicks, E. L. Dewald, H. F. Robey, J. R. Rygg, N. B. Meezan, M. J. Rosenberg, H. G. Rinderknecht, S. Friedrich, R. Bionta, R. Olson, J. Atherton, M. Barrios, P. Bell, R. Benedetti, L. Berzak Hopkins, R. Betti, D. Bradley, D. Callahan, D. Casey, G. Collins, S. Dixit, T. Döppner, D. Edgell, M. J. Edwards, M. Gatu Johnson, S. Glenn, S. Glenzer, G. Grim, S. Hatchett, O. Jones, S. Khan, J. Kilkenny, J. Kline, J. Knauer, A. Kritcher, G. Kyrala, O. Landen, S. LePape, C. K. Li, J. Lindl, T. Ma, A. Mackinnon, A. Macphee, M. J.-E. Manuel, D. Meyerhofer, J. Moody, E. Moses, S. R. Nagel, A. Nikroo, A. Pak, T. Parham, R. D. Petrasso, R. Prasad, J. Ralph, M. Rosen, J. S. Ross, T. C. Sangster, S. Sepke, N. Sinenian, H. W. Sio, B. Spears, P. Springer, R. Tommasini, R. Town, S. Weber, D. Wilson, and R. Zacharias, *Phys. Plasmas* **21**, 112701 (2014).
- A. B. Zylstra, J. A. Frenje, F. H. Séguin, J. R. Rygg, A. Kritcher, M. J. Rosenberg, H. G. Rinderknecht, D. G. Hicks, S. Friedrich, R. Bionta, N. B. Meezan, R. Olson, J. Atherton, M. Barrios, P. Bell, R. Benedetti, L. Berzak Hopkins, R. Betti, D. Bradley, D. Callahan, D. Casey, G. Collins, E. L. Dewald, S. Dixit, T. Döppner, M. J. Edwards, M. Gatu Johnson, S. Glenn, G. Grim, S. Hatchett, O. Jones, S. Khan, J. Kilkenny, J. Kline, J. Knauer, G. Kyrala, O. Landen, S. LePape, C. K. Li, J. Lindl, T. Ma, A. Mackinnon, M. J.-E. Manuel, D. Meyerhofer, E. Moses, S. R. Nagel, A. Nikroo, T. Parham, A. Pak, R. D. Petrasso, R. Prasad, J. Ralph, H. F. Robey, J. S. Ross, T. C. Sangster, S. Sepke, N. Sinenian, H. W. Sio, B. Spears, R. Tommasini, R. Town, S. Weber, D. Wilson, C. Yeaman, and R. Zacharias, *Phys. Plasmas* **22**, 056301 (2015).
- A. B. Zylstra, J. A. Frenje, M. Gatu Johnson, G. M. Hale, C. R. Brune, A. Bacher, D. T. Casey, C. K. Li, D. McNabb, M. Paris, R. D. Petrasso, T. C. Sangster, D. B. Sayre, and F. H. Séguin, *Phys. Rev. Lett.* **119**, 222701 (2017).
- M. Gatu Johnson, A. B. Zylstra, A. Bacher, C. R. Brune, D. T. Casey, C. Forrest, H. W. Herrmann, M. Hohenberger, D. B. Sayre, R. M. Bionta, J.-L. Bourgade, J. A. Caggiano, C. Cerjan, R. S. Craxton, D. Dearborn, M. Farrell, J. A. Frenje, E. M. Garcia, V. Y. Glebov, G. Hale, E. P. Hartouni, R. Hatarik, M. Hohensee, D. M. Holunga, M. Hoppe, R. Janezic, S. F. Khan, J. D. Kilkenny, Y. H. Kim, J. P. Knauer, T. R. Kohut, B. Lahmann, O. Landoas, C. K. Li, F. J. Marshall, L. Masse, A. McEvoy, P. McKenty, D. P. McNabb, A. Nikroo, T. G. Parham, M. Paris, R. D. Petrasso, J. Pino, P. B. Radha, B. Remington, H. G. Rinderknecht, H. Robey, M. J. Rosenberg, B. Rosse, M. Rubery, T. C. Sangster, J. Sanchez, M. Schmitt, M. Schoff, F. H. Séguin, W. Seka, H. Sio, C. Stoeckl, and R. E. Tipton, *Phys. Plasmas* **24**, 041407 (2017).
- M. G. Johnson, D. T. Casey, M. Hohenberger, A. B. Zylstra, A. Bacher, C. R. Brune, R. M. Bionta, R. S. Craxton, C. L. Ellison, M. Farrell, J. A. Frenje, W. Garbett, E. M. Garcia, G. P. Grim, E. Hartouni, R. Hatarik, H. W. Herrmann, M. Hohensee, D. M. Holunga, M. Hoppe, M. Jackson, N. Kabadi, S. F. Khan, J. D. Kilkenny, T. R. Kohut, B. Lahmann, H. P. Le, C. K. Li, L. Masse, P. W. McKenty, D. P. McNabb, A. Nikroo, T. G. Parham, C. E. Parker, R. D.

- Petrasso, J. Pino, B. Remington, N. G. Rice, H. G. Rinderknecht, M. J. Rosenberg, J. Sanchez, D. B. Sayre, M. E. Schöff, C. M. Shulberg, F. H. Séguin, H. Sio, Z. B. Walters, and H. D. Whitley, *Phys. Plasmas* **25**, 056303 (2018).
- ¹⁷G. Maynard and C. Deutsch, *Phys. Rev. A* **26**, 665 (1982).
- ¹⁸G. Maynard and C. Deutsch, *J. Phys.* **46**, 1113–1122 (1985).
- ¹⁹C. Li and R. Petrasso, *Phys. Rev. Lett.* **70**, 3059 (1993).
- ²⁰C.-K. Li and R. D. Petrasso, *Phys. Rev. Lett.* **114**, 199901 (2015).
- ²¹L. S. Brown, D. L. Preston, and R. L. Singleton, Jr., *Phys. Rep.* **410**, 237–333 (2005).
- ²²C.-K. Li and R. D. Petrasso, *Phys. Rev. Lett.* **70**, 3063–3066 (1993).
- ²³D. O. Gericke and M. Schlages, “Beam-plasma coupling effects on the stopping power of dense plasmas,” *Phys. Rev. E* **60**, 904–910 (1999).
- ²⁴A. B. Zylstra, J. A. Frenje, P. E. Grabowski, C. K. Li, G. W. Collins, P. Fitzsimmons, S. Glenzer, F. Graziani, S. B. Hansen, S. X. Hu, M. G. Johnson, P. Keiter, H. Reynolds, J. R. Rygg, F. H. Séguin, and R. D. Petrasso, *Phys. Rev. Lett.* **114**, 215002 (2015).
- ²⁵A. B. Zylstra, J. A. Frenje, P. E. Grabowski, C. K. Li, G. W. Collins, P. Fitzsimmons, S. Glenzer, F. Graziani, S. B. Hansen, S. X. Hu, M. G. Johnson, P. Keiter, H. Reynolds, J. R. Rygg, F. H. Séguin, and R. D. Petrasso, *J. Phys.: Conf. Ser.* **717**, 012118 (2016).
- ²⁶J. A. Frenje, P. E. Grabowski, C. K. Li, F. H. Séguin, A. B. Zylstra, M. Gatu Johnson, R. D. Petrasso, V. Y. Glebov, and T. C. Sangster, *Phys. Rev. Lett.* **115**, 205001 (2015).
- ²⁷J. A. Frenje, R. Florido, R. Mancini, T. Nagayama, P. E. Grabowski, H. Rinderknecht, H. Sio, A. Zylstra, M. Gatu Johnson, C. K. Li, F. H. Séguin, R. D. Petrasso, V. Y. Glebov, and S. P. Regan, *Phys. Rev. Lett.* **122**, 015002 (2019).
- ²⁸D. B. Sayre, C. J. Cerjan, S. M. Sepke, D. O. Gericke, J. A. Caggiano, L. Divol, M. J. Eckart, F. R. Graziani, G. P. Grim, S. B. Hansen, E. P. Hartouni, R. Hatarik, S. P. Hatchett, A. K. Hayes, L. F. B. Hopkins, M. G. Johnson, S. F. Khan, J. P. Knauer, S. L. Pape, A. J. MacKinnon, J. M. McNaney, N. B. Meezan, H. G. Rinderknecht, D. A. Shaughnessy, W. Stoeffl, C. B. Yeaman, A. B. Zylstra, and D. H. Schneider, *Phys. Rev. Lett.* **123**, 165001 (2019).
- ²⁹W. Cayzac, V. Bagnoud, M. M. Basko, A. Blažević, A. Frank, D. O. Gericke, L. Hallo, G. Malka, A. Ortner, A. Tauschwitz, J. Vorberger, and M. Roth, *Phys. Rev. E* **92**, 053109 (2015).
- ³⁰W. Cayzac, A. Frank, A. Ortner, V. Bagnoud, M. Basko, S. Bedacht, C. Bläser, A. Blažević, S. Busold, O. Deppert, J. Ding, M. Ehret, P. Fiala, S. Frydrych, D. Gericke, L. Hallo, J. Helfrich, D. Jahn, E. Kjartansson, A. Knetsch, D. Kraus, G. Malka, N. Neumann, K. Pépitone, D. Pepler, S. Sander, G. Schaumann, T. Schlegel, N. Schroeter, D. Schumacher, M. Seibert, A. Tauschwitz, J. Vorberger, F. Wagner, S. Weih, Y. Zobus, and M. Roth, *Nat. Commun.* **8**, 15693 (2017).
- ³¹C. Li, *Conceptual Study of Moderately-Coupled Plasmas and Experimental Comparison of Laboratory X-Ray Sources* (MIT, 1993).
- ³²B. Trubnikov, *Particle Interactions in a Fully Ionized Plasma* (Consultant’s Bureau, 1965).
- ³³H. G. Rinderknecht, M. J. Rosenberg, A. B. Zylstra, B. Lahmann, F. H. Séguin, J. A. Frenje, C. K. Li, M. Gatu Johnson, R. D. Petrasso, L. F. Berzak Hopkins, J. A. Caggiano, L. Divol, E. P. Hartouni, R. Hatarik, S. P. Hatchett, S. Le Pape, A. J. MacKinnon, J. M. McNaney, N. B. Meezan, M. J. Moran, P. A. Bradley, J. L. Kline, N. S. Krashennnikova, G. A. Kyrala, T. J. Murphy, M. J. Schmitt, I. L. Tregillis, S. H. Batha, J. P. Knauer, and J. D. Kilkenny, *Phys. Plasmas* **22**, 082709 (2015).
- ³⁴A. Hayes, G. Jungman, A. Schulz, M. Boswell, M. M. Fowler, G. Grim, A. Klein, R. S. Rundberg, J. B. Wilhelmy, D. Wilson *et al.*, *Phys. Plasmas* **22**, 082703 (2015).
- ³⁵R. Drake, *High-Energy-Density Physics* (Springer, 2006).
- ³⁶P. E. Grabowski, M. P. Surh, D. F. Richards, F. R. Graziani, and M. S. Murillo, *Phys. Rev. Lett.* **111**, 215002 (2013).
- ³⁷J. Ziegler, J. Biersack, and U. Littmark, *The Stopping and Range of Ions in Matter* (Pergamon, New York, 1985).